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STOCHASTIC MODELS IN THE MAINTENANCE MANAGEMENT

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Abstract*:*
Technical system operation is a stochastic process that can be divided into states of operation. This process is the complex of events that happen to the system. It includes the storage, usage, maintenance, repair and ageing of system. These modes of existence are called states of operation. If states of operation are defined according to their means well-separated, this process can be modeled by a discrete state-space Markov models. Using these models, the work expenditure of operation can be investigated. The presentation deals with Markov model of technical system operation process and describes a possibility of its use.

1. Introduction

Technical system operation and maintenance is a stochastic process that can be divided into states of operation. This process is the complex of events that happen to the system from manufacturing to its rejection. It includes the storage, usage, maintenance and repair of system. These modes of existence are called states of operation. If states of operation are defined according to their means well-separated, the process can be modeled by discrete state-space Markov process (in other words Markovian random walk) (Rohács & Simon 1989).

Stochastic processes whose development in the future is influenced by their development in the past only through their development in the present, that is stochastic processes without after-effects, are called Markov-processes (Wentzel & Ovcharov 1986). The history of studying such processes started with the activity of Andrei Andreievitch Markov (1856-1922) the Russian mathematician.

Many papers have appeared dealing with Markov models for the optimal maintenance decision.

The mathematical basis of Markov process theory and its application can be known by books of Bharucha-Reid (Bharucha-Reid 1960) and Wentzel and Ovcharov (Wentzel & Ovcharov 1986).

Rohács shows several case studies to demonstrate possibilities of use of Markov process theory for investigation of aircraft operational systems and processes (Rohács & Simon 1989).

Song considers the problem of production and preventive maintenance control in a stochastic manufacturing system (Song 2009).

By Dimitrakos and Kyriakidis, the Markov decision process is a mathematical model, which is used to describe a stochastic process controlled by a sequence of actions. They developed an efficient semi-Markov decision algorithm, which operates on the class of control limit policies (Dimitrakos & Kyriakidi 2008).

Guo and Yang presents a new technique that does not use any intermediate model to automatically create Markov models in order to assess the reliability measurements of safety instrumented systems (Guo & Yang 2008).

The author studied possibilities of use of Markov process theory to investigate military aircraft maintenance processes (Pokorádi 1994a), (Pokorádi 1994b), (Pokorádi 1997), (Pokorádi 1998), (Pokorádi 2002), (Pokorádi 2008).

The main goals of paper are to show Markov models of technical system maintenance, and a maintenance management method based upon the Markovian random walk process.

The rest of the paper is organized as follows. The definition of the Markov processes is given in Section 2. In Section 3, we present the general model of ageing processes of technical systems. The Section 4 shows a maintenance management methods based on Markov model, theoretically. The Section 5 presents a short case study by former scientific work of the author. In the last section, the conclusions of this work are given

2. The Markov-Processes

The mathematically described random process $\eta(t)$ is called Markov-one if the equation of conditional probabilities.

$$P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_1) = X_1 \ldots \eta(t_n) = X_n\right) = P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_n) = X_n\right) \quad (1)$$

proves to be true with the probability 1 for each $t_1 < \ldots < t_n < t_{n+1}$ and $X_1; X_2; \ldots; X_i; X_{i+1}$ real numbers.

If process $\eta(t)$ during the study period can have an *X* value at any moment, it is called a continuous-time process. If $\eta(t)$ can only have some value at certain moments, the process is called a discrete-time one. A random process is considered to be of discrete state space, if the possible values of variance $\eta(t)$ constitute a finite set or a count non-finite set.

Finite or count non-finite stochastic processes, that is the discrete state space ones with no after-effects, are called Markov-chain (Wentzel & Ovcharov 1986). In this case, the value established in the equation (1) is called the transition probability:

$$P_{ij}^{n,n+1} = P\left(\eta(t_{n+1}) = X_{n+1} \middle| \eta(t_n) = X_n\right) \quad . \quad (2)$$

The transition probability expresses that $\eta(t_{n+1}) = X_j$, supposing that $\eta(t_n) = X_i$.

$P_{ij}^{n,n+1}$ marking above also shows that the transition probability is the function of not only the *i*-th beginning state and of the *j*-th next state, but it is also the function of t_n time.

A Markov-process can be characterized unambiguously by supplying the transition probabilities, and the distributions of leaving different states. If distribution of leaving different states are not of the exponential character, such stochastic process is called Semi-Markov one.

3. The Ageing Process

During the operation, a technical system wears out by stochastic effects. Therefore its technical state goes through continuous, cumulative stochastic changes. In case of the usage the technical state generally changes in a negative sense, while during the maintenance or repair, it changes positively.

For demonstration, let general parameter η characterize the technical state of the investigated system (see figure 1). If the value of parameter η meets the η_{br} brake value, the system breakdowns. Let τ be the parameter, which characterizes the performance of the system. For example, this parameter can be the effective calendar time, effective operating hours, kilometers from installation or the last overhaul.

In this case the wearing-out process of the system, that is the $\eta(\tau)$ stochastic function can be characterized by:

$\hat{\eta}(\tau)$ expected value function of the parameter η;

$f(\eta,\tau)$ density function of parameter η.

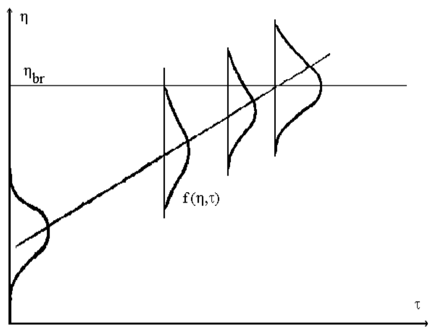


Figure 1 The Aging Process

Then the probability of good working state of the system:

$$P_{gw}(\tau) = P(\eta_{br} > \eta(\tau)) = \int_{-\infty}^{\eta_{br}} f(\eta,\tau)d\tau \quad . \qquad (3)$$

The process of changing of parameter η can be describe by:

$\overset{\circ}{\eta}(\tau)$ changing velocity of the parameter η;

$\varphi\left(\overset{\circ}{\eta},\tau\right)$ density function of the changing velocity.

Then the "failure changing velocity" of parameter η is:

$$\overset{\circ}{\eta}_{br}(\tau,\Delta\tau)=\frac{\eta_{br}-\eta(\tau)}{\Delta\tau} \qquad \text{if} \qquad \eta_{br}>\eta(\tau) \tag{4}$$

and the probability of good working state of the system in the interval (η , $\eta+\Delta\eta$):

$$P_{gw}(\tau,\Delta\tau)=P\left(\overset{\circ}{\eta}_{br}(\tau)>\overset{\circ}{\eta}(\tau)\right)=\int_{-\infty}^{\overset{\circ}{\eta}_{br}}\varphi\left(\overset{\circ}{\eta},\tau\right)d\overset{\circ}{\eta} \quad ,. \tag{5}$$

supposing that the system is ready to service at the start of the investigated performance interval.

4. Maintenance Management Method

Depending on the momentary values of the leader parameter and its velocity, the needed service work can be decided. For decision, permissible value and velocity of the leader parameter should be determined on the basis of its breakdown value and permissible probability of risk.

Knowing the breakdown value η_{br} of the parameter η and performance interval between checks $\Delta\tau$, the permissible value η_p and permissible changing velocity to ready for working should be determined. Supposing that:

- the change of the parameter η on interval $\Delta\tau$ (see figure 2) is a linear one;
- density function of the changing velocity is independent on working time of the system.
- In this case, if value of the parameter η reaches the permissible value η_p at the *i*-th checking and it changes with

$$\overset{\circ}{\eta}>\frac{\Delta\eta}{\Delta\tau}$$

velocity, the parameter η is going to reach breakdown value η_{br} before next ($i+1$-th) check, in the other words the operated system will break-down.

Therefore, permissible velocity of parameter η to ready for working is:

$$\overset{\circ}{\eta}_p=\frac{\Delta\eta}{\Delta\tau} \quad . \tag{6}$$

The probability of breakdown is:

$$P_{br}(\Delta\tau,\Delta\eta)=P\left(\overset{\circ}{\eta}>\overset{\circ}{\eta}_{br}\right)=1-P\left(\overset{\circ}{\eta}\le\overset{\circ}{\eta}_{br}\right)=1-\int_{-\infty}^{\overset{\circ}{\eta}_{br}}\varphi(\overset{\circ}{\eta})d\overset{\circ}{\eta} \tag{7}$$

Knowing permissible probability of risk Q (permissible probability of breakdown), it is substituted into equation (5), equation

$$Q = P_{br}(\Delta\tau, \Delta\eta) = 1 - \int_{-\infty}^{\mathring{\eta}_p} \varphi(\mathring{\eta})\, d\mathring{\eta} \qquad (8)$$

is got.

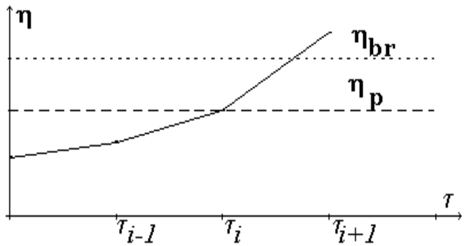


Figure 2 Determination of the Permissible Parameter Values

If the density function of velocity $\mathring{\eta}$ cannot be determined by statistical method, usage of one of known density functions is suitable. For example:

- UNIFORM distribution:

$$\varphi(\mathring{\eta}) = \frac{1}{\mathring{\eta}_{max} - \mathring{\eta}_{min}} = \frac{1}{\Delta\mathring{\eta}} \quad (\text{if } \mathring{\eta}_{max} > \mathring{\eta} > \mathring{\eta}_{min}) \quad . \qquad (9)$$

Then

$$Q = 1 - \int_{-\infty}^{\mathring{\eta}_p} \frac{1}{\Delta\mathring{\eta}} d\mathring{\eta} = 1 - \frac{\mathring{\eta}_p}{\Delta\mathring{\eta}} = 1 - \frac{\Delta\eta}{\Delta\tau\Delta\mathring{\eta}} \quad , \qquad (10)$$

that is

$$\Delta\eta = (1 - Q)\Delta\tau\Delta\mathring{\eta} \quad . \qquad (11)$$

- EXPONENTIAL distribution:

$$\varphi(\mathring{\eta}) = \lambda e^{-\lambda\eta} \quad (\text{if } \mathring{\eta} > 0) \quad . \qquad (12)$$

Then

$$Q = 1 - e^{-\lambda \overset{\circ}{\eta}_p} \quad , \tag{13}$$

and

$$\Delta\eta = -\frac{\ln(1-Q)}{\lambda}\Delta\tau \tag{14}$$

✈ NORMAL (GAUSS) distribution:

$$\varphi(\overset{\circ}{\eta}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\overset{\circ}{\eta}-m)^2}{2\sigma^2}} \quad . \tag{15}$$

In this case simply solution cannot be got like to above ones, which is deduced easily in algebraic way. Therefore, on the basis of its variance and expected value, transforming the normal distribution to the standard normal one, the permissible velocity of parameter η and parameter interval $\Delta\eta$ can be determined.

The permissible value of the parameter η to ready for working:

$$\eta_p = \eta_{br} - \Delta\eta \quad . \tag{16}$$

If momentary values η and $\overset{o}{\eta}$ smaller that those determined by equation (16) and (6), the system will not break down till the next check with probability of least $1-Q$.

5. A Short Example of the Method's Usage

For demonstrating the possibility of use of above mentioned method, the setting up and usage of mathematical model of aircraft brake-system will be shown.

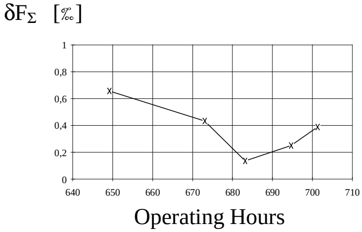


Figure 3 Decrease of the Brake-Effort Depending on Operating Hours

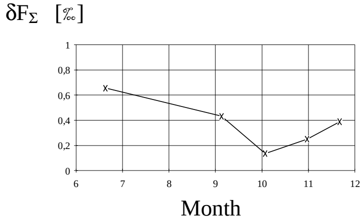


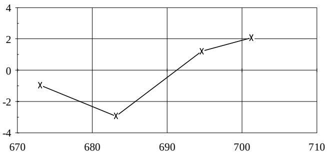
Figure 4 Decrease of the Brake-Effort Depending on Calendar Time (of the Investigating Year)

The decrease of the brake-effort and brake asymmetry was chosen as leader parameters. To determine the permissible value and velocity of this leader parameters,

$$Q = 0{,}025$$

permissible probability of risk was used.

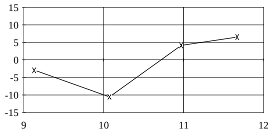
$\frac{d\delta F_{\Sigma}}{dt_w}$ [f hour^{-5}]



Operating Hours

Figure 5 Changing Velocity of the Brake-Effort Depending on Operating Hours

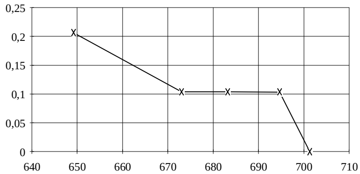
$\frac{d\delta F_{\Sigma}}{dt_c}$ [day^{-6}]



Month

Figure 6 Changing Velocity of the Brake-Effort Depending on Calendar Time (of the Investigating Year)

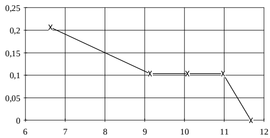
δF_{Δ} [‰]



Operating Hours

Figure 7 Brake-Asymmetry Depending on Operating Hours

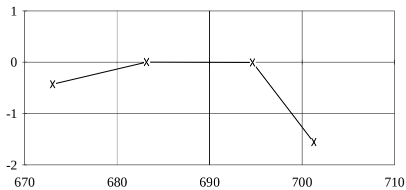
δF_{Δ} [‰]



Month

Figure 8 Brake-Asymmetry Depending on Calendar Time (of the Investigating Year)

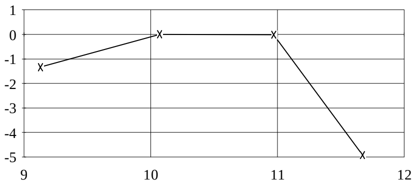
$\frac{d\delta F_{\Delta}}{dt_w}$ [f.hour^{-5}]



Operating Hours

Figure 9 Changing Velocity of Brake-Asymmetry Depending on Operating Hours

$\frac{d\delta F_{\Delta}}{dt_c}$ [day^{-6}]



Month

Figure 10 Changing Velocity of Brake-Asymmetry Depending on Calendar Time (of the Investigating Year)

The quantity of data is not sufficient for statistical estimation of their distribution. Therefore, for determination of the permissible value and velocity of the resultant brake-effort, the densities of its changing velocity is supposed as uniform one.

6. Conclusions

The paper shown the Markovian models of technical system operational (maintenance) processes shortly In this study the management method of technical operation, as a Markovian random walk process, has been formulated in case of aircraft break system. Technical data needed for usage of mathematical diagnostics and operational management method can be obtained by using gages of the helicopter and instrument of technical service team. The shown maintenance management method is able to minimization of technical service work with adequate safety of operation. The possibility of use of shown method has been proven by examination of pneumatic brake-system a regular aircraft.

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