Pokorádi L., Uncertainties of Engineering Simulation, Proceedings of International Conference on Innovative Technologies IN-TECH 2010, Prague, Czech Republic, 14.09.2010. – 16.09.2010. (ISBN 978-80-904502-2-6), p. 121–124.

POSTPRINT

Uncertainties of Engineering Simulation

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Keywords: modeling, simulation, model uncertainty

Abstract. During mathematical simulation of a real technical system we can meet any type and rate model uncertainty. Its reasons can be incognizance of modelers or data inaccuracy. On the one hand the real technical systems are precise, but complex. Additionally the large-scale systems consist of large number of inter-related subsystems. On the other hand, the mathematical model should be simple therefore may be imprecise. Mathematical modeling and simulation of complex technical systems must include the nondeterministic features of the modeled system and the environment of the system. These nondeterministic features mean that the response of the system cannot be predicted precisely because of the existence of uncertainty in the system or the environment. The paper will give a short overview of types and sources of model uncertainties and illustrates model uncertainty examination methods.

Introduction

The main application of mathematics in the engineering is the mathematical modeling and simulation. Using mathematical models, technical system analysis or synthesis can be done. During mathematical modeling the modeler can meet any type and rate model uncertainty. Its reasons can be incognizance of modelers or data inaccuracy.

Oberkampf encouraged a dialog between the risk assessment, reliability engineering, and generalized information theory communities on the subject of uncertainty representation, aggregation, and propagation [1]. In another paper, Oberkampf demonstrated a new framework of the general phases of modeling and simulation by a system-level example: the flight of a rocket-boosted. aircraft-launched missile [2].

Möller and Beer discussed non-probabilistic uncertainty modeling is by means of interval modeling and fuzzy methods [3].

Perincherry et al. proposed an approach to analyze uncertainties of large-scale systems. This approach enables to represent the context depended nature of uncertainty [4].

Ferson and Tucker shown the relationship between Probability Bounds Analysis (PBA) and methods of interval analysis and probabilistic uncertainty analysis from which it is jointly derived, and indicated how the method can be used to assess the quality of probabilistic models such as those developed in Monte Carlo simulations for risk analyses. They also illustrated how a sensitivity analysis can be conducted within a PBA by pinching inputs to precise distributions or real values. [5].

Sebastian and Ledoux worded, the key challenges in preliminary design are reliability and robustness [6]. The robustness of a product can be defined as the capacity of a product to maintain its performance values in spite of the changing in functioning conditions and the variability or the uncertainties of its parameters. In preliminary design context, sources of uncertainties can be distinguished in two different classes: (i.) the aleatory or random uncertainty, and (ii.) the epistemic uncertainty.

Kiureghian and Ditlevsen discussed the sources and characters of uncertainties in engineering modeling for risk and reliability analyses [7]. In their paper two simple examples demonstrated the influence of statistical dependence arising from epistemic uncertainties on systems and time-variant reliability problems.

Rohács showed the problems of flight dynamics and control [8]. Rohács represented, that the

mathematical model of the aircraft motion contains the uncertainties that can be characterized in the following ways:
- noise and disturbances;
- parametric uncertainty;
- unmodeled dynamics.

Szabolcsi derived mathematical model of the aeroelastic aircraft fuselages bending motion using slender beam theory, and modeled the bending motion as additive parameter uncertainty affecting motion of the rigid body aircraft [9].

The author applied interval [10] and probabilistic [11] methods to investigate effects of manufacturing parameter uncertainties. The author also used second-order probability uncertainty investigation method and worked up a new two-dimensional probabilistic maintenance estimation method theoretically, and shown possibility of use and experiments of first application of developed method [12].

Thee paper gives a short overview of the types and sources of model uncertainties and illustrates model uncertainty examination methods by literatures mentioned above.

The paper will be organized as follows: Section 1 shows the applied literatures. Section 2 words the types of model uncertainties. Section 3 presents the investigation methods of parametrical uncertainties. Section 5 illustrates methodology of linear interval analysis method's usage to investigate effects of manufacturing parameter uncertainties.

Types of Model Uncertainties

The mathematical model is the mathematical equation or system of equations which describes the internal principles of the process occurring on the system from the point of view of the given investigation. On the one hand, the real systems are precise but complex. Additionally the large-scale systems consist of large number of inter-related subsystems. On the other hand, the mathematical model should be simple therefore may be imprecise. Mathematical modeling and simulation of complex technical systems must include non-deterministic features of the modeled system and its environment or human interaction with the system. These non-deterministic features mean that the response of the system cannot be predicted precisely because of the existence of uncertainty in the system or the environment.

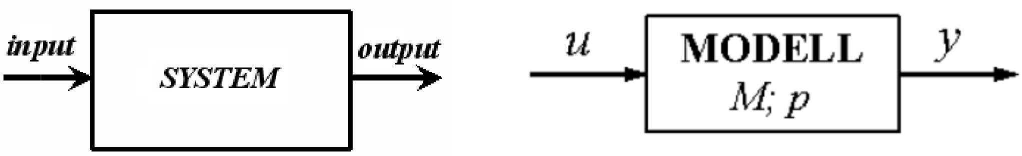

Fig. 1. System and Model

Seeing Fig. 1., a mathematical model has

M	—	structure (e.g. linear, stochastic, discrete time etc.);
p	—	inner system parameters (e.g. proportional control factor, gear ratio, constants etc.);
u	—	input signs (e.g. control signals, ambient parameters etc.);

and responds by y output parameters. For interpretation of types of uncertainty and their investigation methods, let

$$\underline{y} = f(\underline{x}) \quad (1)$$

general mathematical model, where $\underline{y}$ is the vector of dependent (output) variables, $\underline{x}$ is the vector of independent variables, which consists of p inner and u input parameters.

One of the most widely recognized distinctions in uncertainty types is between *aleatory* and *epistemic* ones.

Uncertainties are characterized as epistemic, if the modeler sees a possibility to reduce them by gathering more data or by refining models. Epistemic uncertainty derives from some level of ignorance of the physical process, the system or the environment. Experts use the term epistemic uncertainty to describe any lack of knowledge or information in any phase of the modeling and model application. This type of uncertainty may be comprised of substantial amounts of both objectivity and subjectivity. Some of types of epistemic uncertainty sources that can occur in technical systems simulation include:

- false knowledge of system or its environment;
- incorrect application of scientific laws;
- selecting the appropriate model formulation;
- model generalization;
- model reduction;
- linearization.

The epistemic uncertainty means the incorrection of M model structure (Fig. 2.a).

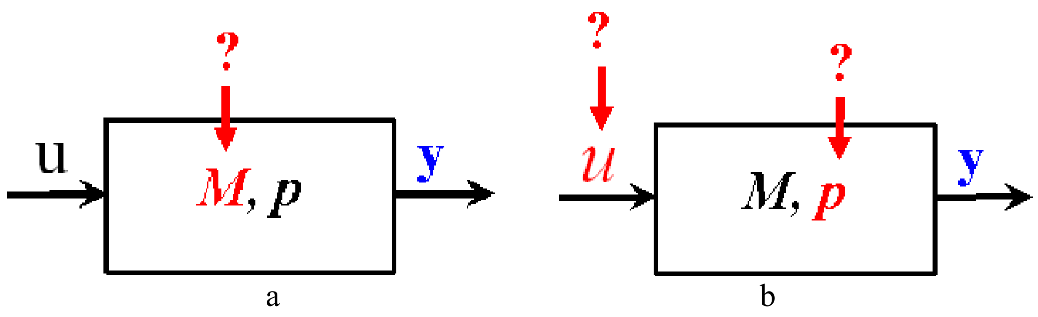

Fig. 2. Epistemic and Aleatory Uncertainty

Aleatory uncertainty is an inherent variation associated with the investigated system or its environment. It is also called as variability, irreducible, random uncertainty or (in the control theory) parametric one. Aleatory uncertainty is primarily associated with objectivity. Its possible „engineering“ sources:

- incorrect measuring;
- measuring noises;
- discretization;
- strong statistical information;
- sparse statistical information;
- using of linguistic data;
- selecting the appropriate database;
- manufacturing anomalies.

The parametric uncertainty means anomalies of parameters p and u (see Fig.2.b). Refrerring to Eq. (1), this type uncertainty is the doubtfulness or fluctuation of vector $\underline{x}$, which can be characterized by

$$\underline{d}_x^{\,T} = \left[d_{x_1}; d_{x_2} \ldots d_{x_n} \right] \tag{2}$$

vector of independent variables' distributions, and/or

$$\underline{i}_x^{\,T} = \left[i_{x_1}; i_{x_2} \ldots i_{x_n} \right] \tag{3}$$

vector of independent variables' intervals.

Methods of Parametrical Uncertainty's Investigation

In uncertainty analysis, a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. By Ferson and Tucker's papers, there are two disparate ways to affect such a study.

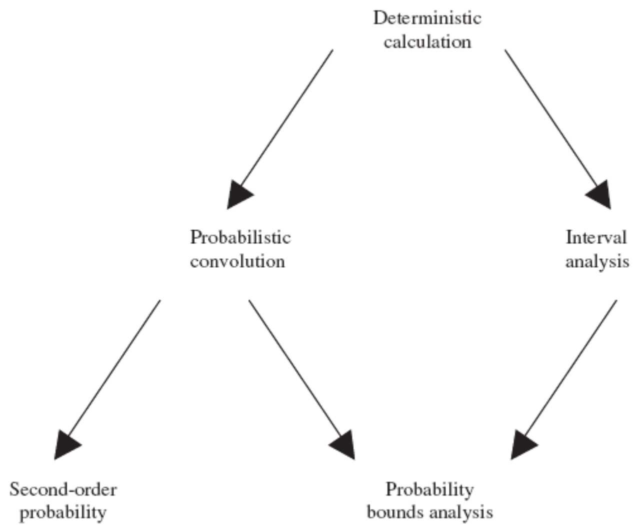

Fig. 3. Relationships among Different Uncertainty Investigation Methods [5]

One of them is the interval uncertainty analysis (see Fig. 3.). During this investigation generally the equation

$$\underline{i}_y = f_i(\underline{i}_x) \tag{4}$$

is used, where the function f_i should be determined by function f of Eq. (1).

Another "natural" uncertainty investigation method is to describe a probability distribution to the elements in the neighborhood. In this case the

$$\underline{d}_y = f_d(\underline{d}_x) \tag{5}$$

equation should be used, where the function f_d describes the connection between distributions of dependent and independent parameters, which should be determined by function f of Eq. (1).

One of the most well-known probabilistic uncertainty investigation methods is the Monte Carlo simulation. The "classical" Monte Carlo simulation is used as an uncertainty analysis of a deterministic calculation because it yields a distribution describing the probability of alternative possible values about the nominal (designed) point.

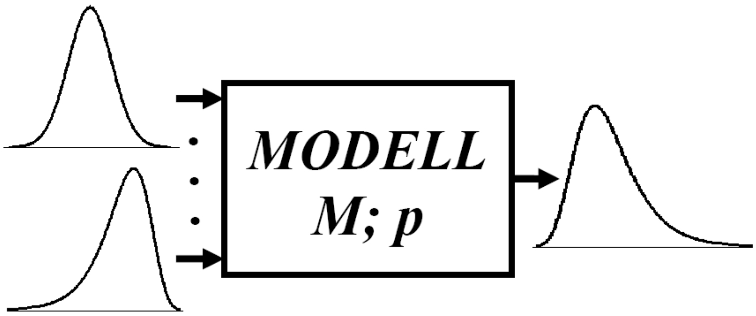

Fig. 4. Monte-Carlo Simulation

From practical engineering point of view, probabilistic uncertainty investigation set two problems. Firstly, the parameters of probability distributions of independent variables can be determined by any statistical method, therefore they have so-called second-order uncertainties. Secondly, from mathematical point of view, the domain of variability of probability distributions is basically infinity.

In the Fig. 3. there are two possible paths are shown as right and left downward arrows.

To answer the first question mentioned above, the left one shows a probabilistic uncertainty analysis of a probabilistic calculation. The resulting analysis would be a second-order probabilistic assessment. For this investigation, the following general form equation should be solved:

$$\underline{d}_y = f_{dd}\left(\underline{d}_{d_x}\right) \tag{7}$$

where

$$\underline{d}_{d_x}{}^T = \left\lfloor d_{d_{x_1}}; d_{d_{x_2}} \ldots d_{d_{x_n}} \right\rfloor \tag{8}$$

is the vector of distribution of independent variables' stochastic parameters and f_{dd} is the function which describes the connection between distributions of independent variables' stochastic parameters and distributions of dependent ones. This function should be determined by function f of Eq. (1) – depend on the main aim of investigation.

Another derived method applies bounding arguments to the probabilistic calculation and arrive at interval versions of probability distributions. The second problem mentioned above can be answered by this method. Therefore the infinity of variability of probability distributions is impossible in engineering practice. Ferson and Tucker call such calculations PBA. This approach represents the uncertainty about a probability distribution by the set of cumulative distribution functions. PBA is an uncertainty analysis of a probabilistic calculation because it defines neighborhoods of probability distributions. During the PBA, the

$$\begin{bmatrix} \underline{d}_y \\ \underline{i}_y \end{bmatrix} = f_{di}\left(\begin{bmatrix} \underline{d}_x \\ \underline{i}_x \end{bmatrix}\right) \tag{6}$$

General form equation should be used, where the function f_{di} describes the connection between distributions and intervals of dependent and independent parameters, which should be determined by function f of Eq. (1).

Summary, Future Work

The author of this paper would like to arouse readers' interest in importance and possibilities of use of mathematical model uncertainty analysis. The paper has shown types of mathematical model uncertainties and the model uncertainty investigation methods shortly.

During prospective scientific research related to this field of applied mathematics, technical system modeling and process simulation, the author would like to work out:

I. methods to depict model uncertainty
  - if uncertainties if p inner system parameters are not independent stochastic variables;
  - using interval linear systems;
  - using fuzzy set theory and fuzzy linear systems.

II. similar examples to demonstrate every uncertainty investigation methods for students.

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