

# ICMA 2012

**The 13th International Conference  
on Mathematics and its Applications**

**Department of Mathematics  
"Politehnica" University of Timișoara**

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**Consilier editorial:** Prof.dr.ing. Sabin IONEL

**Redactor:** Claudia MIHALI

**Bun de imprimat:** 03.04.2013

**Coli de tipar:** 22,5

**ISSN** 1224-6069

Tiparul executat la S.C. U.R.C. XEDOS S.R.L.

# FUZZY RULE BASED RISK ASSESSMENT WITH SUMMARIZED DEFUZZIFICATION

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## Abstract

Nowadays, a lot of fuzzy rule based risk assessment methods are in practice. For the lot of ones there are not able to handle the extremeness according to the authors because of some kind of averaging dissolve the extremeness immediately at the beginning and it just goes on with average especially if extremeness is high or low in opinions or in measured data. The opinion or the experiences of experts are important for risk assessment and it should be taken into consideration the extremenesses as well but not as an average at the beginning. So the authors emphasizes a new method to solve the problem.<sup>1</sup>

## 1 Introduction

The motivation was to handle the extremenesses, which containing important informations about an investigated area. At a company when one do any kind of questionnaires for investigating some area one do not have too much data from fulfilled questionnaires. Therefore one can not take immediately the input data as average at least at the beginning because information are lost.

Ross (2010) has a well done introduction into fuzzy logic and its engineering applications in fuzzy decision process as well. Pokorádi (2008) investigated fuzzy decision process in engineering. Liu et al. (2010) did a literature review about putting fuzzy mathematics into practice on risk assessment. Ross et al. (2002) show the deference between fuzzy logic and probability. Johanyák (2012) shows the clonal selection algorithm which was tested successfully.

The aim of the paper is to offer a new fuzzy rule based risk assessment method. The Summarized DeFuzzyfication (SDF) can handle the extremenesses of data in fuzzy decision process.

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<sup>1</sup>Mathematical Subject Classification(2008):03E72, 03E75

Keywords and phrases:*fuzzy decision, defuzzyfication, summarized defuzzyfication*

The outline of the paper is as follows: The Section 2. shows the classical fuzzy decision process with a sample. The Section 3. represents the method of summarized defuzzification theoretically and provides a sample for it. The Section 3. goes on summary and future work and shows the acknowledgment. The last section gives the references.

## 2 The traditional fuzzy rule based decision

The classical fuzzy decision method has the next parts: fuzzyfication, inference, composition and defuzzification. In fuzzyfication part one put the input data to fuzzyfy them for fuzzy decision process. The next part is the inference in which If-Then rules are created for decision process. The composition is the next one. Here one should handle the multi-value of same fuzzy membership functions and the last part is the defuzzification to have crisp value for the fuzzy decision process. All of these are shown on Figure 1.

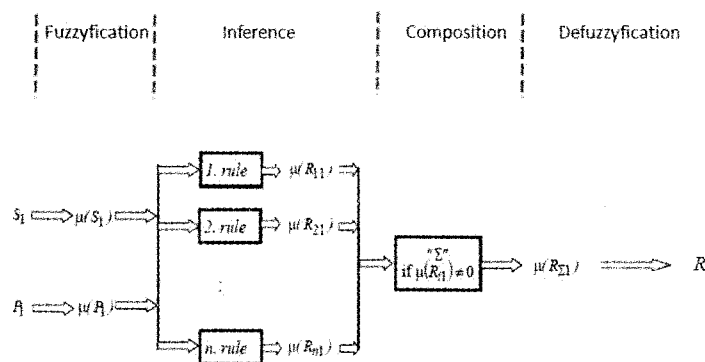


Figure 1: Traditional fuzzy rule based decision flowchart

The center of gravity (COG) is used in this paper for defuzzification method according to equation (1).

$$Y_{COG} = \frac{\sum_{i=1}^n \int_{-\infty}^{\infty} \mu_i(x) \cdot x dx}{\sum_{i=1}^n \int_{-\infty}^{\infty} \mu_i(x) dx} \quad (1)$$

- $Y$  – the crisp value of defuzzification,
- $\mu_i(x)$  – the  $i^{th}$  fuzzy membership function,
- $n$  – the number of fuzzy membership functions.

### 2.1 Traditional sample

At this point, the Authors give a sample to demonstrate the classical fuzzy rule based decision.

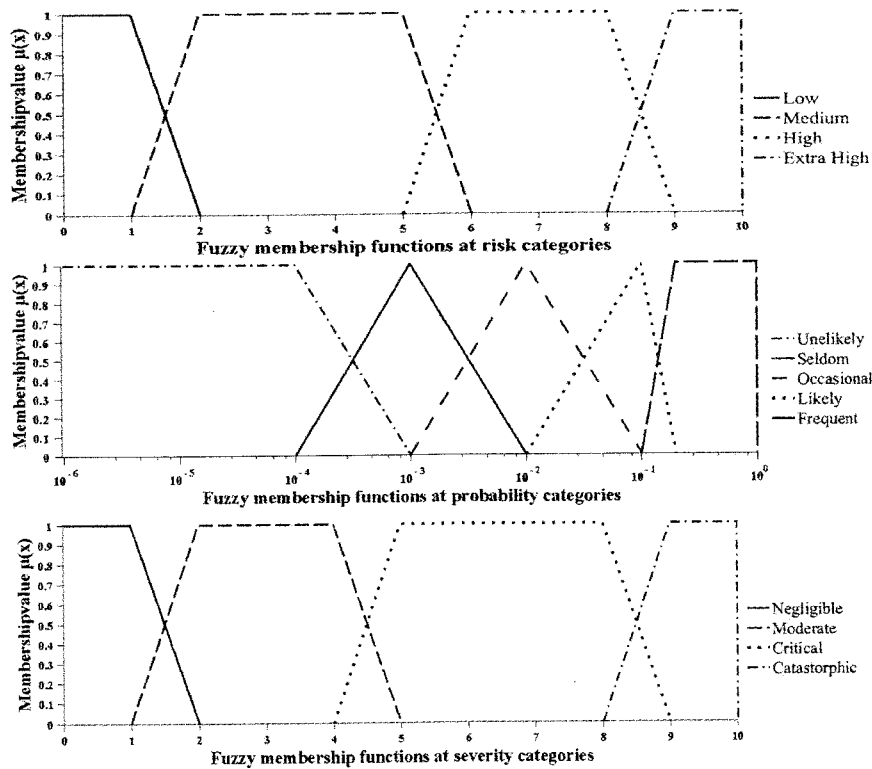


Figure 2: Membership function at different categories

|              | Frequent | Likely | Occasional | Seldom | Unlikely |
|--------------|----------|--------|------------|--------|----------|
| Catastrophic | EH       | EH     | H          | H      | M        |
| Critical     | EH       | H      | H          | M      | L        |
| Moderate     | H        | M      | M          | L      | L        |
| Negligible   | M        | L      | L          | L      | L        |

EH – Extra High, H – High, M – Medium, L – Low

Table 1: The Risk Assessment Matrix

First of all the severity, the probability and the risk linguistic variables are defined. For each value of fuzzy linguistic variables belongs a fuzzy membership function. The severity linguistic variable has the next values like negligible, moderate, critical and catastrophic and defined on Figure 2. The probability linguistic variable has the next values like unlikely, seldom, occasional, likely and frequent and defined on Figure 2. The risk linguistic variable has the next values like low, medium, high, extra high and defined on Figure 2.

The next step is to create the If-Then rules which represent in this case in a so-called risk assessment matrix on Table 1. Now let it be given the X's opinion about severity with 4.5 and X's opinion about the probability with 0.006 and Y's opinion of severity with 5 and Y's opinion of probability with 0.004. Their average of severity is 4.75 and their average of probability is 0.005. These are the input data.

In condition of If-Then rules are the fuzzyfied input data which join together with minimum operator. The inference has the next non zero results for risk:

If the severity is Critical with 0.75 and the probability is Occasional with 0.699 then the risk is High with 0.699.

If the severity is Moderate with 0.25 and the probability is Occasional with 0.699 then the risk is Medium with 0.25.

If the severity is Critical with 0.75 and the probability is Seldom with 0.301 then the risk is Medium with 0.301.

If the severity is Moderate with 0.25 and the probability is Seldom with 0.301 then the risk is Low with 0.25.

There are the input data fuzzyfied in the conditions of If-Then rules and the conclusion parts contain the membership values of the risks. In the enumeration of If-Then rules, there are two Medium linguistic values for risk linguistic variable with 0.301 and 0.25 fuzzy membership values. These two medium values are joint with maximum operator in composition. If a linguistic variable has more same linguistic values then one, so these same linguistic values are joint together with maximum operator. Therefore the medium value is 0.301 for risk. So the crisp value of risk is 5.14 using the equation ( 1 ). This result can be given to experts or managers to make their decisions because they do responsibility for that.

### 3 Summarized DeFuzzyfication (SDF)

This section introduces shortly the theoretical description of summarized defuzzyfication thereafter gives a sample for it. The theoretical consideration is to can be handle the extremenesses before the averaging of input data. To count the fuzzy rule based decision process one must be to have data from questionnaires.

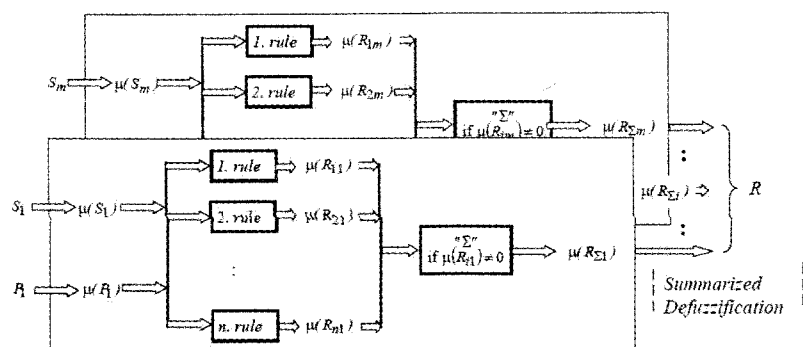


Figure 3: Fuzzy risk with summarized defuzzyfication flowchart

At the beginning, all input data without any averaging must be let till the composition part also the composition must be done too. So the result of decision is in fuzzy form for all input data. Thereafter one must do the defuzzyfication process

for crisp value with the equation ( 2 ) . The summarized defuzzification is shown on Figure 3.

$$Y_{SCOG} = \frac{\sum_{i=1}^n \sum_{j=1}^m \int_{-\infty}^{\infty} \mu_{ij}(x) \cdot x \, dx}{\sum_{i=1}^n \sum_{j=1}^m \int_{-\infty}^{\infty} \mu_{ij}(x) \, dx} \quad (2)$$

- $Y_{SCOG}$  – the crisp value of summarized defuzzification,  
 $\mu_{ij}(x)$  – the  $j^{th}$  fuzzy membership function  
 which belongs to  $i^{th}$  opinion (input data),  
 $n$  – the number of opinions (input data),  
 $m$  – the number of fuzzy membership functions.

### 3.1 Sample for summarized defuzzification

At this point the created model in 2.1 subsection is usable for this sample. It means the severity, the probability and the risk categories with their fuzzy membership functions are the same and the risk matrix as well. The input data are the same. Now the X's fuzzy opinion about the risk is Low with 0.222, Medium with 0.5 and High with 0.5 from where the crisp value is counted with equation ( 1 ) which is 4.644. After the Y's fuzzy opinion about the risk is Medium with 0.398 and High with 0.602 from where the crisp value is counted with equation ( 1 ) which is 5.346. Now the common fuzzy opinion is counted according to equation ( 2 ) where the number of fuzzy membership function  $m = 4$  also the number of risk categories furthermore the number of opinions  $n = 2$ . So the result for common fuzzy opinion is **4.972**. The common fuzzy opinion is on Figure 4. That result is quite different from the averaging one.

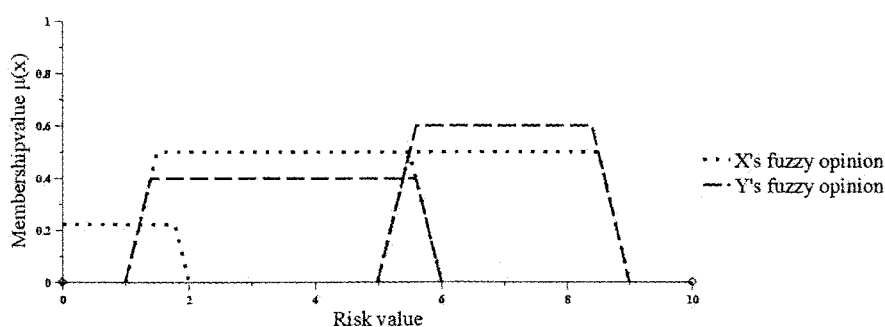


Figure 4: Result at “Summarized” composition

## 4 Summary and future work

The summarized defuzzification method is sensitive to extreme expert opinion. The summarized defuzzification takes the extremenesses much better into consideration than the traditional defuzzification method because it does not skip the extremenesses but it allows to go along the fuzzy decision process till the composition and on the last step takes into consider the extremenesses at defuzzification step. The managers and/or experts must have responsibilities about using results based upon their knowledge and experiences. The Authors future work is to put into practice SDF and test the method.

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## Acknowledgment

The work/publication is supported by the TÁMOP-4.2.2/B-10/1-2010-0024 project. The project is co-financed by the European Union and the European Social Fund.

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