ICMA 2012

The 13th International Conference on Mathematics and its Applications

Department of Mathematics "Politehnica" University of Timişoara

Editors:
Ioan CUCULESCU
Jovo JARIĆ
Paşc GĂVRUŢA
Ioan GOLEŢ
Liviu CĂDARIU

EDITURA POLITEHNICA

EDITURA POLITEHNICA
Bd. Republicii nr. 9
300159 Timişoara, România

Tel./Fax. 0256/403.823
E-mail: editura@edipol.upt.ro

Consilier editorial: Prof.dr.ing. Sabin IONEL
Redactor: Claudia MIHALI

Bun de imprimat: 03.04.2013
Coli de tipar: 22,5
ISSN 1224-6069

Tiparul executat la S.C. U.R.C. XEDOS S.R.L.

Proceedings of the 13th Conference on Mathematics and its Applications
University "Politehnica" of Timisoara
November, 1-3, 2012

MODULAR FAULT TREE SENSITIVITY ANALYSIS

László POKORÁDI
University of Debrecen

Abstract

The Probabilistic Fault Tree Analysis (PFTA) is a quantitative method used to calculate the probability of the system failure from given failure probabilities of system components. The objective of the sensitivity analysis is to show how the change in any system parameter influences the resultant reliability value of the whole system. The main aim of this study is to elaborate an easy-used modular approach Linear Fault Tree Sensitivity Model (LFTSM). [1]

1 Introduction

Reliability analysis is an important aspect of designing and evaluating safety critical and fault-tolerant systems. When the results of a reliability analysis are generated, a common set of questions result. Which component or module is contributing the most to the unreliability of the system? How much do our results depend on the accuracy of the input parameters to the system reliability? The answers to these questions require analysis of sensitivity of the reliability results (Ou & Dugan (2000)).

The Wua et al's paper establishes the fault tree analysis models of a solar array mechanical system and analyzes reliability to find mechanisms of the solar array fault (Wua et al. (2011)).

There are lot of methods to investigate reliability of integrated systems. One of them is the Fault Tree Analysis (FTA) The FTA is a systematic, deductive (top-down type) and probabilistic risk assessment tool which shows the causal relations leading to a given undesired event, referred to as the "Top Event" (TE). The events, which cannot be subdivided, are called the Basic Events (BEs). FT diagram displays the undesired state of the investigated system (TE) in terms of the states of its

[1]Mathematical Subject Classification(2008):*49Q12, 94C12,90B25*
Keywords and phrases:*sensitivity analysis, fault tree analysis, reliability, LFTSM*

components (BEs). The FTA is a graphical design technique the main result of wich is a graph that has a dendritic structure (Pokordi (2011)). A method of FT sensitivity analysis is shown in paper of Csiba by an example of the complete railway vehicle. The reliability model of the railway carriage has been built by 16 main constructional units (Csiba (2008)).

The Wua et al's paper establishes the fault tree analysis models of a solar array mechanical system and analyzes reliability to find mechanisms of the solar array fault (Wua et al. (2011)).

The main aim of this study is to show an easy-usable algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM). This new modular sensitivity model, investigation method uses matrix-algebraic mathematical tool that is based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines (Pokorádi (2008)).

The outline of the paper is as follows: Section 2 shows theoretically the modular sensitivity investigation algorithm. Section 3 discusses the Fault Tree Analysis and Linear Fault Tree Sensitivity Model. Section 4 interprets an application of the LFTSM. Section 5 summaries the paper, outlines the prospective scientific work of the Author.

2 Modular Sensitivity Analysis

The sensitivity analysis shows how sensitive the output parameter with respect to changing in any element of the input parameters? During modular sensitivity analysis firstly the system should be separated into modules. Investigating these modules, their models should be determined by $y_j = f(x_1, x_2, \ldots, x_n)$ general form, where $n \in \mathbb{N}$ is number of input parameters. If we look at instead of y_j a function $f(x_1, x_2, \ldots, x_n)$ the parameter vector $\mathbf{x}$, the sensitivity coefficient K_{y_j,x_i} can be simply computed for y depending on x_i (Pokorádi (2008)):

$$K_{y_j,x_i} = \frac{\partial y}{\partial x_i}\frac{x_i}{y}, K_{y_j,x_i} \subset \mathbb{R}. \tag{1}$$

Using these coefficients, the relative changing depending on relative changing of vector's $\mathbf{x}$ elements of y can be determined by

$$\delta y_j = K_{y_j,x_1}\delta x_1 + \ldots + K_{y_j,x_n}\delta x_n. \tag{2}$$

equation, where: δ signs the relative changing of parameters. Next task is to separate system parameters into vectors $\mathbf{y}$ dependent and $\mathbf{x}$ independent ones. Then, the connection between relative changing of system parameters can be described by

$$\mathbf{A}\delta\mathbf{y} = \mathbf{B}\delta\mathbf{x}. \tag{3}$$

where: $\mathbf{A} \in I\!R^{n \times n}$ and $\mathbf{B} \in I\!R^{n \times m}$ are coefficient matrices of dependent and non dependent variables of the investigated system. Using the

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B}, \mathbf{D} \in I\!R^{n \times m}. \tag{4}$$

where: $m \in I\!N$ – number of output parameters.

Relative Sensitivity Coefficient Matrix, the equation

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x}. \tag{5}$$

can be used for relative sensitivity investigations. This matrix equation describes the relative sensitivities of investigated system. The i-th row of matrix $\mathbf{D}$ shows how sensitivity of input variables of the i-th system module.

3 Fault Tree Analysis and the LFTSM

A FTA is basically a logic tree that shows the association between events on the tree. In general, two forms of logic gates appear on a FT: the **AND** and the **OR** gate. The **AND** logical gate should be used if output event occurs only if all input events occur simultaneously. If output event occurs if any of the input events occur, either alone or in any combination, the **OR** logical gate should be used. The probabilities of "output events", and the sensitivity coefficients of i-th input events:

In case of **AND** logical gate:

$$P = \prod_{i=1}^{n} P_i \ , \ K_i = 1. \tag{6}$$

where:
P_i , $P_i \in [0, 1] \subset I\!R$ – probability of occurrence of i-th input event;
n , $n \in I\!N$ – number of input events.
In case of **OR** logical gate:

$$P = 1 - \prod_{i=1}^{n} P_i \ , \ K = \frac{P_j}{P} \prod_{i=1}^{n} (1 - P_i). \tag{7}$$

Next task is to separate events of Fault Tree into Basic Events and Non-Basic (Top and Intermediate) Events (NBEs). The probabilities should be arranged into vectors $\mathbf{x}$ (BEs) and $\mathbf{y}$ (NBEs), and sensitivity coefficients into matrices $\mathbf{B}$ (BEs) and $\mathbf{A}$ (NBEs). Using the method shown by Chapter 2, the Relative Sensitivity Coefficient Matrix of FTA $\mathbf{D}$ can be got. The first row of matrix $\mathbf{D}$ will be used as Relative Sensitivity Coefficient Vector of TE $\mathbf{d} \in I\!R^{1 \times n}$.

4 Case Study

To demonstrate the setting-up methodology mentioned above, lets' study the Fault Tree shown by Figure 1. In the figure the events signed by numbers are Intermediate Events (IEs), and the events signed by letters are BEs. The Table 1. shows the nominal values of Bacis Events' probabilities. The vectors of probabilities of BEs, and NBEs are:

$$\begin{aligned} \mathbf{x}^T &= [P_A; P_B; P_C; P_D; P_E; P_F; P_G; P_H; P_I; P_J; P_K; P_L; P_M; P_N; P_O; P_P; P_R]; \\ \mathbf{y}^T &= [P_{TE}; P_1; P_2; P_{11}; P_{12}; P_{13}; P_{21}; P_{22}; P_{111}; P_{211}]; \end{aligned} \tag{8}$$

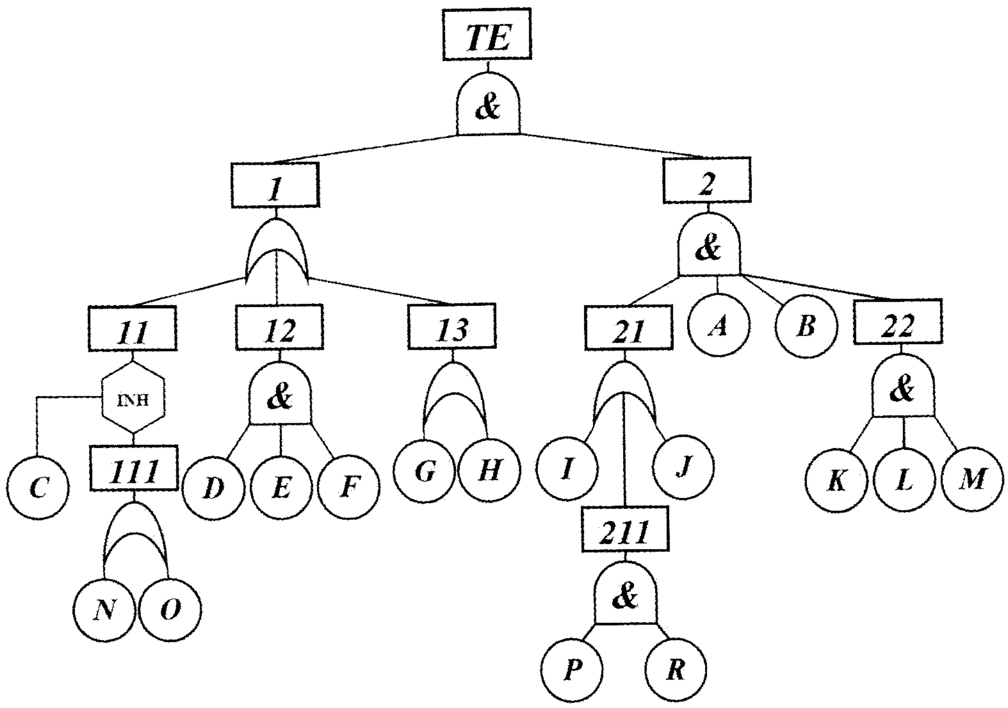

Figure 1: Fault Tree

A	4.8010^{-4}	G	3.4010^{-4}	M	1.5010^{-4}
B	9.2010^{-4}	H	3.4010^{-7}	N	8.9010^{-5}
C	2.3010^{-4}	I	3.6010^{-5}	O	8.9010^{-5}
D	5.7210^{-7}	J	1.4010^{-4}	P	1.9010^{-5}
E	1.1210^{-7}	K	2.4010^{-4}	R	1.9010^{-5}
F	1.1210^{-7}	L	3.5010^{-4}		

Table 1: Nominal Values of BEs' Probabilities

The coefficient matrices of BEs, and NBEs are:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -K_{11} & -K_{12} & -K_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -K_{211} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$\mathbf{B} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&K_G&K_H&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&K_I&K_J&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&K_N&K_O&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1 \end{bmatrix} \tag{10}$$

According to vector **d** which is viewable on equation (11), the following conclusions can be drawn:

- the A; B; K; L and M BEs have the highest;
- the D; E and F BEs have the least

influence on TE's probability. These conclusions can be used to determine technical tasks to improve reliability of investigated system. These questions go beyond of this study.

$$\mathbf{d}^T = [1.0, 1.0, 0.369, 7.8110^{-5}, 7.8110^{-5}, 7.8110^{-5}, 0.513, 0.513, \\ 0.163, 0.870, 1.0, 1.0, 1.0, 0.085, 0.086, 0.146, 0.146]\,; \tag{11}$$

5 Conclusions

This paper discussed the LFTSM elaborated by the Author. The advantages of the modular approach method shown above are followings: The connections between input and output event of all logical gates can be written easily. Therefore, employing these equations, the sensitivity coefficients of all logical gates as modules of investigated FT that is elements of coefficient matrices can be determined easily. After determination of required vectors and matrices, the experts can use matrix-algebraic method to analyze sensitivity characters of given FT from given investigational point of view.

References

[1] J. Csiba, Sensitivity Analysis of the Reliability Computed by Using the Failure Tree Method,*Proc. of the* 11^{th} *Mini Conference on Vehicle System Dynamics, Identification and Anomalies*, Budapest, 749-760, (2008)

[2] Y. Ou, J. B. Dugan, Sensitivity Analysis of Modular Dynamic Fault Trees *Proc. Computer Performance and Dependability Symposium*, IPDS 2000, IEEE International, 35 - 43 (2000)

[3] L. Pokorádi, "Systems and Processes Modeling", Campus Kiadó, Debrecen, (in Hungarian) (2008)

[4] L. Pokorádi, Sensitivity Investigation of Fault Tree Analysis with Matrix-Algebraic Method.*Theory and Applications of Mathematics & Computer Science* Vol. 1, 34-44, (2011)

[5] J. Wua, S.Yan, L. Xie, Reliability analysis method of a solar array by using fault tree analysis and fuzzy reasoning Petri net, *Acta Astronautica* 69 960-968 (2011)

Acknowledgment

The publication is supported by the TÁMOP-4.2.2/B-10/1-2010-0024 project. The project is co-financed by the European Union and the European Social Fund.

The publication was supported by the TÁMOP-4.2.2.C-11/1/KONV-2012-0001 project. The project has been supported by the European Union, co-financed by the European Social Fund.

SZÉCHENYI PLAN

László Pokorádi
University of Debrecen,
Ótemető út 2-4., H-4028, Debrecen, Hungary
E-mail: pokoradi@prosysmod.hu