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Graph Theoretical Investigation of Network Structure System

Pokorádi László

Abstract: During modeling of real technical systems or processes, it is a key issue the determination of existence of interconnection between the components or states. In cases of integrated system its exposure can be difficult task because of complexity of interconnections. The aim of the paper is to show an easy-usable algorithm for determination of existence of interconnection between the network structure system-components or states of technical processes.

Keywords: graph theory; network structure system;

1 INTRODUCTION

During analysis or synthesis of integrated, network structure technical systems, it is an important key issue to determine existence of interconnection between the system components.

Graphs are models; to study interconnections between components of network structure systems and technical (for example maintenance) processes. Graph theory is the historical mathematical foundation of the modern network's science. The "Bridges of Königsberg" problem, first proposed by Leonhard Euler, is analyzed using basic properties of graphs. As it turns out, the first graph in graph theory history is impossible to traverse without traversing one or more links repeated times, because the degree of all nodes in the Bridges of Königsberg graph are odd-valued. Thus, a traveler must visit at least one bridge twice, to cover the entire graph [4].

Adrásfavi's book shows the mathematical basis of graph theory [1]. The mathematical background of this study can be read in handbook of Bronstejn [2] and textbook of Fazekas [3]. Lewis' book contains theoretical knowledge and practical possibilities of use of network science to investigate networks and network structure systems [4]. Pokorádi presented the mathematical models and their application in the engineering [5].

This paper is aimed to show an easy-usable algorithm for determination of existence of interconnection between the system-components or states of technical processes, in other words to determine the connection matrix knowing the adjacent matrix.

The structure of this paper is as follows: Section 1 contains the applied literatures and the main goals of investigation. Section 2 presents the theoretical background of graph theory shortly. Section 3 shows a matrix algebraic method to determine connection matrix by a case study of a manufacturing plant. A summation, conclusion and the extension of the topic in the future will be given in Section 4.

2 THEORETICAL BACKGROUNDS

A graph G = [N,L,f] is a 3-tuple consisting of a set of nodes N, a set of links L, and a mapping function f : $L \rightarrow N \times N$, which maps links into pairs of nodes. Nodes directly connected by a link are called adjacent nodes.

When the node-pair order does not matter in linking the node pair, G is an undirected graph.

In an undirected graph G, $p_i \sim p_j$ is equivalent to $p_j \sim p_i$. Figure 1 visualizes the topology of undirected graph G [5].

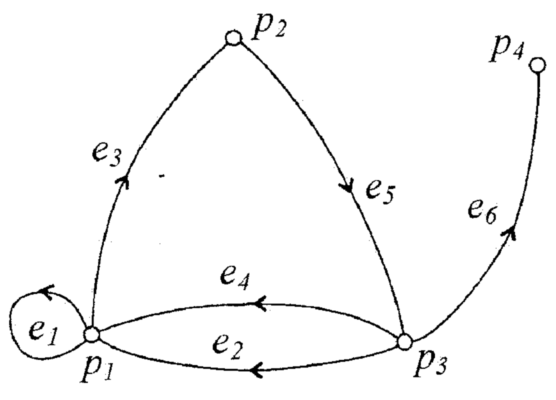

Fig. 1. An undirected graph

G' (see Figure 2.) is a directed graph because $p_i \rightarrow p_j$ is not $p_j \rightarrow p_i$. Commuting a node pair does make a difference in the graph's topology. Links are directed, by definition, in a directed graph. A link defined by the node pair

(p_i ; p_j) is not the same as a link defined by node pair (p_j ; p_i). In fact, both links may exist in a directed graph.

Set theory is especially efficient for proving theorems and rigorously defining the properties of a graph, but for some forms of analysis a matrix representation is more effective.

A graph's mapping function f can be represented by the Adjacency matrix A, and the connection matrix Z.

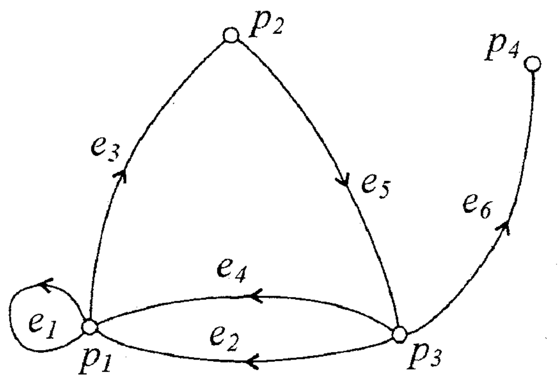

Fig. 2. A directed graph

The direct links are ignored in the graph's adjacency matrix A — which show the number of links directly connecting node i to node j. This number is stored at row i, column j of the matrix.

The adjacent matrix of undirected graph G shown by Figure 1.:

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \qquad (1)$$

The adjacent matrix of directed graph G' shown by Figure 2.:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (2)$$

The connection matrix Z contains a 1 in row i column j if one or more links connect node i to node j. In other words, connection matrix shows that can we get at node j from node i. For example, the connection matrix can be used for troubleshooting or to determine "sink stetas" of technical processes in engineering practice.

The length of a path is equal to the number of links between starting and ending nodes of the path.

The longest path between any two nodes in a graph G is called the diameter of G that is denoted as:

$$\text{diameter}(G) \ .$$

The longest path from a node u to all other nodes of a connected graph be defined as the radius of node u that is denoted as:

$$\text{radius}(u) \ .$$

The node(s) with the largest radius, are called peripheral nodes.

A path that begins and ends with the same node is called a circuit.

3 DETERMENATION OF CONNECTION MATRIX

In case of big, size, integrated technical system it is a real practical problem to expose connection of system elements, id est to determine the connection matrix lightly. Therefore an easy-used method is proposed which calculates connection matrix Z from adjunct one A.

To demonstrate the matrix algebraic method let us investigate a manufacturing plant shown by Figure 3.

Firstly the adjunct matrix should be determined. This task is an easy one, because considering the system the neighboring aggregates and their direct interconnection can be seen easily. In case of the adjacent matrix of investigated system is shown by Equation (3)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (3)$$

It is obvious that $a_{ij}^{[k]}$ element of the k-th power matrix A^k of adjacent one A shows the number of independent k-long paths from i-th to j-th node of the graph. (Precise and exact mathematical proof of the proposition mentioned above can be seen in book of Fazekas [3].)

The element $h_{ij}^{[k]}$ of

$$H_k = \sum_{n=1}^{k} A^n \ . \qquad (4)$$

summarized mtrix of power matrices shows the number of independent, maximum k-long paths from i-th to j-th node of the graph.

Let us generate signum matrix S_k of H_k, using following function:

$$S_k = \text{sign } H_k$$
$$s_{ij}^{[k]} = \text{sign } h_{ij}^{[k]} \ , \qquad (5)$$

where:

$$\text{sign}\,\eta = \begin{cases} 1, & \text{if } \eta > 0 \\ 0, & \text{if } \eta = 0 \\ -1, & \text{if } \eta < 0 \end{cases} . \qquad (6)$$

The element $s_{ij}^{[k]}$ of this obtained matrix shows existence of maximum k-long connection from i-th to j-th nodes.

It is obvious too, that the longest path (circuit which connects all nodes) of graph equal number of its nodes denoted N.

So, the connection matrix of a graph which has N nodes can be determined by the following equation:

$$Z = \text{sign}\sum_{n=1}^{N} A^{n} \; . \qquad (7)$$

In case of investigated system shown by Figure 3., the connection matrix:

$$Z = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \qquad (8)$$

The deducing technical conclusions from the Equation (8) goes beyond of this study.

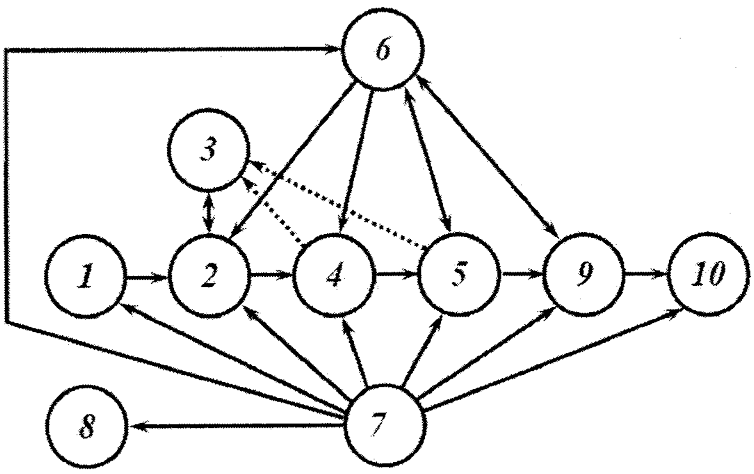

Fig. 3. Graph of Investigated System
1 – charging of source material; 2 – preparatory treatment; 3 – waste product; 4 – molding-machine; 5 – drying machine; 6 – conditioning of humidity and temperature; 7 – heat-supplier; 8 – exhaust products; 9 – burner; 10 – packaging department.

4 CONCLUTIONS

The paper showed an easy-usable algorithm to determine connection between the network structure system's components or states of technical processes. Using the proposed method can determine the connection matrix of investigated system if its adjacent matrix is known.

During prospective scientific research related to this field of the applied mathematics and the science of engineering management, the author would like to develop other models and methods to investigate engineering systems

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