PROCEEDINGS

of the

13th MINI CONFERENCE ON VEHICLE SYSTEM DYNAMICS, IDENTIFICATION AND ANOMALIES

Held at the

Faculty of Transportation Engineering
Budapest University of Technology and Economics, Hungary
BUDAPEST, 5-7 November, 2012

Edited by Prof. I. Zobory

VSDIA 2012

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

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The Organising Committee thanks the authors for their efforts in providing the electronic version of the full paper manuscripts. Issued by the ***Department of Railway Vehicles and Vehicle System Analysis at the BME***, Budapest, 2012. **ISBN 978-963-313-102-2**

LINEAR FAULT TREE SENSITIVITY MODEL

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Received: November 7, 2012

ABSTRACT

The Fault Tree Analysis (FTA) is a systematic, deductive (top-down type) and probabilistic risk assessment tool which shows the causal relations leading to a given undesired event, referred to as the "Top Event" (TE). The events, which cannot be subdivided, are called the Basic Events (BE). Fault Tree diagram displays the undesired state of the investigated system (Top Event) in terms of the states of its components (BEs). The Fault Tree Analysis is a graphical design technique main result of which is a tree, a dendritic graph. Probabilistic Fault Tree Analysis (PFTA) is a quantitative analysis method used to calculate the probability of Top Event from given failure probabilities of system components. The objective of the sensitivity analysis is to show how the change in any system parameter influences the resultant reliability value of the whole system. The main aim of this study is to elaborate an easy-used algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM). This new modular approach process uses matrix-algebraic method based upon the mathematical diagnostic modeling of aircraft systems and gas turbine engines. The paper shows the adaptation of linear mathematical diagnostic modeling methodology for setting-up of LFTSM and its possibility of use to investigate Fault Tree sensitivity by an example.

Keywords: Fault Tree Analysis; sensitivity analysis; LFTSM

1. INTRODUCTION

The Fault Tree Analysis (FTA) is considered one of the more useful analytical tools in the system safety process, especially when evaluating extremely complex or detailed systems. The FTA is a very organized, meticulous, and versatile type of analysis. It is organized because it evaluates each event in consideration of that event's specific purpose, function, or place within a system or process. The FTA is meticulous because it attempts to describe the relationship of any and all events that may have acted on a system to result in the top event. This method is also quite versatile in its ability to allow for the evaluation of hypothetical events, which the analyst may introduce into the tree to determine potential effects on the Top Event (TE) [7].

The Fault Tree Analysis (FTA) is a systematic, deductive (top-down type) and probabilistic risk assessment tool which shows the causal relations leading to a given undesired event, referred to as the "Top Event" (TE). The events, which cannot be subdivided, are called the Basic Events (BEs). FT diagram displays the undesired state of the investigated system (TE) in terms of the states of its components (BEs). The FTA is a graphical design technique the main result of wich is a graph that has a dendritic structure [3].

The objective of the sensitivity analysis is to show how the change in any system parameter influences the resultant reliability value of the whole system.

The paper of Tchorzewska-Cieslak and Boryczko contains the methodology of the FTA and an example of its application in order to analyze different failure scenarios in water distribution subsystem. They concluded that the FTA is particularly useful for the analysis of complex technical systems in which analysis of failure scenarios is a difficult process because it requires to examine a high number of cause-effect relationship. Undoubtedly the water distribution subsystem belongs to such systems.

The FTA involves "thinking back", which allows the identification of failure events that cause the occurrence of the TE. In the case of very large fault trees it is advisable to use the computer methods [6].

An algorithm of vague FTA is proposed in Chang et al.'s paper to calculate fault interval of system components from integrating expert's knowledge and experience in terms of providing the possibility of failure of bottom events. This paper also modifies Tanaka's definition on FTA and integrates vague set arithmetic for implementing fault-tree analysis on weapon system fault diagnosis [1].

Paper of the Author showed an adaptation of linear mathematical diagnostic modeling methodology for setting-up of Linear Fault Tree Sensitivity Model (LFTSM) and its possibility of use to investigate Fault Tree's sensitivity by a demonstrative example [5].

A method of FT sensitivity analysis is shown in paper of Csiba by an example of the complete railway vehicle. The determination of resultant reliability is indispensable for the investigation. The determination of the reliability and the fulfillment of sensitivity analysis will be carried out by using the FT method. In the investigation of Csiba, the reliability of a railway carriage has been determined. The reliability model of the railway carriage has been built by 16 main constructional units [2].

One of the disadvantages of Csiba's work, that only sensitivity of TE's probability can be investigated in case of changing of BEs' probabilities. His method cannot analyze sensitivities of Intermediate Events (IEs), which can represent sensitivity of element groups' or subsystems' reliability. Another disadvantage is that calculation of sensitivity coefficient can be difficult in a real, complicate situation. (It will be shown in Chapter 3 by demonstrative example of this paper)

The main aim of this study is to elaborate an easy-used algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM). This new modular approach process uses matrix-algebraic method that is based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines by the Author [5]. This paper will show adaptation of linear mathematical diagnostic modeling methodology for setting-up of LFTSM and its possibility of use to investigate Fault Tree sensitivity by a demonstrative example.

The outline of the paper is as follows: Section 2 shows a new modular approach algorithm for setting-up of Linear Fault Tree Sensitivity Model (LFTSM) theoretically. Section 3 interprets a simply application of the LFTSM. Section 4 summaries the paper, outlines the prospective scientific work of the Author.

2. THE LINEAR FAULT TREE SENSITIVITY MODEL (LFTSM)

In generally, the question of the sensitivity analysis is how sensitive the output parameter y with respect to changing in any coordinate of the input parameter-vector $\mathbf{x}$? If we look at instead of y a function $f(x_1;\ x_2\ ...\ x_n)$ the parameter vector $\mathbf{x}$, the sensitivity coefficient $K_{y;xi}$ can be simply computed for y depending on x_i [4]:

$$K_{yx_i} = \frac{\partial y}{\partial x_i}\frac{x_i}{y}\ ,\quad K_{yx_i} \subset \Re\ . \qquad (1)$$

Using these coefficients, the relative changing depending on relative changing of vector's **x** elements of *y* can be determined by

$$\delta y = K_{yx_1}\delta x_{yx_2} + \ldots + K_{yx_n}\delta x_n \ . \tag{2}$$

equation, where: δ signs the relative changing of parameters.

The first step in a FTA is the selection of the TE that is a specific undesirable system's state or failure. Then the experts should analyze the system or process to discover logical dependencies between TE and all BEs. To represent logical dependencies, basically the ***AND*** or ***OR*** logical gates and so-called Intermediate Events (IEs) can be used. The IEs can denote subsystem's faults.

The ***AND*** logical gate should be used if output event occurs only if all input events occur simultaneously. If output event occurs if any of the input events occur, either alone or in any combination, the ***OR*** logical gate should be used.

In case of ***AND*** logical gate:

- the probability of "output event":

$$P = \prod_{i=1}^{n} P_i \ , \tag{3}$$

where:

P_i	$P_i \in [0,1] \subset \Re$	—	probability of occurrence of *i*-th input event;
n	$n \in \aleph$	—	number of input events.

- the sensitivity coefficient of *i*-th input event:

$$K_i = 1 \ . \tag{4}$$

In case of ***OR*** logical gate:

- the probability of "output event":

$$P = 1 - \prod_{i=1}^{n} (1 - P_i) \ . \tag{5}$$

- the sensitivity coefficient of *i*-th input event:

$$K_j = \frac{P_j}{P} \prod_{\substack{i=1 \\ i \neq j}}^{n} (1 - P_i) \ . \tag{6}$$

Next task is to separate events of Fault Tree into Basic Events and Non-Basic (Top and Intermediate) Events (NBEs). The probabilities of basic and non-basic events should be arranged into vectors $\mathbf{x}$ and $\mathbf{y}$. Then, the connection between relative changings of probabilities of BEs and NBEs can be described by

$$\mathbf{A}\delta\mathbf{y} = \mathbf{B}\delta\mathbf{x} \ . \tag{7}$$

where: $\mathbf{A} \in \Re^{n \times n}$ and $\mathbf{B} \in \Re^{n \times m}$ are coefficient matrices of BEs and NBEs of the investigated FT. Using the

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B} \quad , \quad \mathbf{D} \in \Re^{n \times m} \ . \tag{8}$$

Relative Sensitivity Coefficient Matrix, the equation

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x} \ . \tag{9}$$

can be used for relative sensitivity investigations. This matrix equation describes the interdependencies between relative changes of BEs and NBEs, the relative sensitivities of investigated system.

The *i*-th row of matrix **D** shows the sensitivity of the *i*-th NBE. So, if only the probability of TE and its sensitivities are investigated, only the first row of matrix **D** will be used as Relative Sensitivity Coefficient Vector of TE $\mathbf{d} \in \Re^{m \times 1}$, and the relative changing of TE probability of occurrence can be calculated by

$$\delta P_{TE} = \mathbf{d}^T \delta\mathbf{x} \ . \tag{10}$$

The advantages of the modular approach method shown above are followings: The connections between input and output event of all logical gates can be written easily — using Equations (4) or (6). Therefore, employing these equations, the sensitivity coefficients of all logical gates as modules of investigated FT that is elements of coefficient matrices **A** and **B** can be determined easily. After determination of required vectors and matrices, the experts can use matrix-algebraic method to analyze sensitivity characters of given FT from given investigational point of view. The next Chapter will show the possibility of use of LFTSM by a real case study.

3. PRACTICAL DEMONSTRATION

To demonstrate the setting-up methodology mentioned above, lets' study the Fault Tree shown by Fig. 1. In the figure the events signed by numbers are Intermediate Events (IEs), and the events signed by letters are BEs.

3.1 The Probabilistic FTA

For further investigation, firstly the probabilities of IEs and at last probabilities of the TE should be determined.

$$P_{TE} = P_1 P_2 \ ; \tag{11}$$

$$P_1 = 1 - \{(1 - P_{11})(1 - P_{12})(1 - P_{13})\} \ ; \tag{12}$$

$$P_2 = P_{21} P_{22} P_A P_B \ ; \tag{13}$$

$$P_{11} = P_{111} P_C \ ; \tag{14}$$

$$P_{12} = P_D P_E P_F \ ; \tag{15}$$

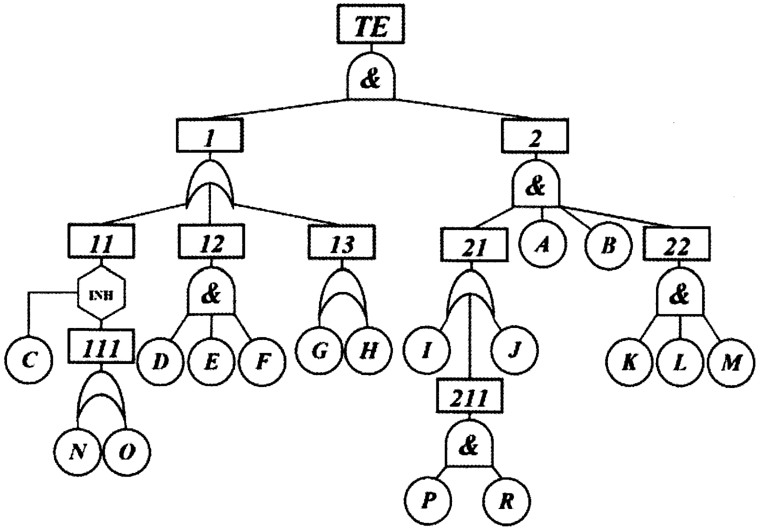

Fig. 1 Fault Tree

$$P_{13} = P_G + P_H - P_G P_H \; ; \tag{16}$$

$$P_{21} = 1 - \{(1 - P_{211})(1 - P_I)(1 - P_J)\} \; ; \tag{17}$$

$$P_{22} = P_K P_L P_M \; ; \tag{18}$$

$$P_{111} = P_N + P_O - P_N P_O \; ; \tag{19}$$

$$P_{211} = P_P P_R \; ; \tag{20}$$

3.2 Sensitivity Model of FTA

The vectors of probabilities of BEs, and NBEs are:

$$\mathbf{x}^T = [P_A; P_B; P_C; P_D; P_E; P_F; P_G; P_H; P_I; P_J; P_K; P_L; P_M; P_N; P_O; P_P; P_R] \; ; \tag{21}$$

$$\mathbf{y}^T = [P_{TE}; P_1; P_2; P_{11}; P_{12}; P_{13}; P_{21}; P_{22}; P_{111}; P_{211}] \; . \tag{22}$$

Using equations (11) — (20), the coefficient matrices of BEs, and NBEs are:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -K_{11} & -K_{12} & -K_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -K_{211} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \tag{23}$$

$$\mathbf{B}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&K_G&K_H&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&K_I&K_J&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&K_N&K_O&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1\end{bmatrix}. \quad (24)$$

The Table 1. shows the nominal values of Bacis Events' probabilities.

A	$4.80\ 10^{-4}$	G	$3.40\ 10^{-4}$	M	$1.50\ 10^{-4}$
B	$9.20\ 10^{-4}$	H	$3.40\ 10^{-7}$	N	$8.90\ 10^{-5}$
C	$2.30\ 10^{-4}$	I	$3.60\ 10^{-5}$	O	$8.90\ 10^{-5}$
D	$5.72\ 10^{-7}$	J	$1.40\ 10^{-4}$	P	$1.90\ 10^{-5}$
E	$1.12\ 10^{-7}$	K	$2.40\ 10^{-4}$	R	$1.90\ 10^{-5}$
F	$1.12\ 10^{-7}$	L	$3.50\ 10^{-4}$		

Table 1. Nominal Values of Bacis Events' Probabilities

The Fig. 2. demonstrates the results of TE sensitivity analysis.

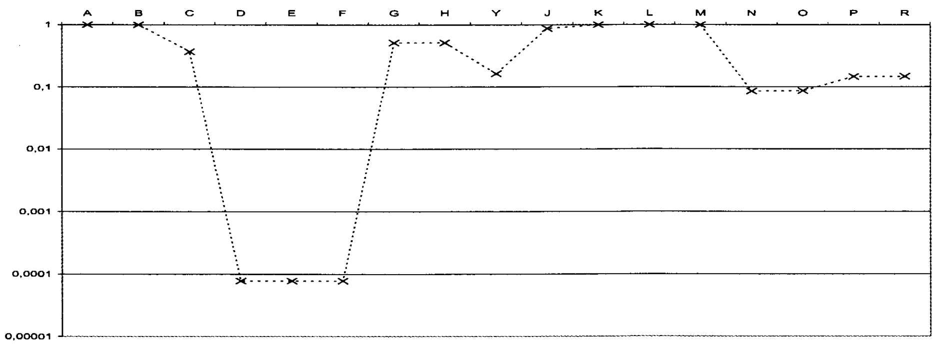

Fig. 2 Result of Sensitivity Analysis

On basis of Fig. 2., the following conclusions can be drawn:

- the *A*; *B*; *K*; *L* and *M* BEs have the highest;
- the *D*; *E* and *F* BEs have the least

influence on TE's probability. These conclusions can be used to determine technical tasks to improve reliability of investigated system. These questions go beyond of this study.

4. CONCLUDING REMARKS

This paper discussed a new sensitivity investigation method of Fault Tree Analysis elaborated by the Author. The paper showed the adaptation of linear mathematical diagnostic modeling methodology for setting-up of Linear Fault Tree Sensitivity Model (LFTSM) The LFTSM is a modular approach tool that uses matrix-algebraic method based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines. In this paper the possibility of use of LFTSM was demonstrated to investigate Fault Tree sensitivity by a simply example. Using demonstrated mathematical connections and procedure, the technical experts can get a easy-used methodology to build up sensitivity model of the given FT. This Linear Fault Tree Sensitivity Model (LFTSM) can be used to investigate system reliability and dependability from required point of view.

During prospective scientific research related to this field of applied mathematics and maintenance management decision making, the Author would like to work out methodologies of Fault Tree uncertainty investigation using other mathematical tools, for example linear interval equations, Monte-Carlo Simulation and fuzzy set theory, on basis of Linear Fault Tree Sensitivity Model (LFTSM).

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6. ACKNOWLEDGEMENT

The publication is supported by the TÁMOP-4.2.2/B-10/1-2010-0024 project. The project is co-financed by the European Union and the European Social Fund.

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